MATH 1251 Mathematics 1B Algebra S2 2008 Test 2 Version 2B (Yellow)

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1. $\mu_1\mathbf{u} + \mu_2\mathbf{v} = \mathbf{0}$ for some μ_1, μ_2 not both zero. Thus the equation $\lambda_1\mathbf{u} + \lambda_2\mathbf{v} + \lambda_3\mathbf{w} = \mathbf{0}$ has a non-zero solution $\lambda_1 = \mu_1$, $\lambda_2 = \mu_2$ and $\lambda_3 = 0$ which by definition means $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly dependent.

2. $\mathbf{0} \in S$ clearly (i.e. the zero polynomial satisfied the condition) and so S is not empty.

   If $p, q \in S$ then $(p+q)(1) + (p+q)(2) = [p(1) + p(2)] + [q(1) + q(2)] = 0 + 0 = 0$ thus S is closed under vector addition.

   If $p \in S$ and $\lambda \in \mathbf{F}$ then $(\lambda p)(1) + (\lambda p)(2) = \lambda p(1) + \lambda p(2) = \lambda\left(p(1) + p(2)\right) = \lambda \times 0 = 0$ and so S is closed under scalar multiplication.

3. (a)
$$\begin{pmatrix} 1 & -1 & 1 & 2 \\ -2 & 3 & 1 & -5 \\ 3 & -2 & 6 & 5 \end{pmatrix} \to \begin{pmatrix} 1 & -1 & 1 & 2 \\ 0 & 1 & 3 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$
   and so a basis for the column space is
$$\left\{ \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}, \begin{pmatrix} -1 \\ 3 \\ -2 \end{pmatrix} \right\}.$$

   (b) Clearly
$$\left\{ \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}, \begin{pmatrix} -1 \\ 3 \\ -2 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}$$
   does the job.

   (c) $T(2,2,2) \neq 2T(1,1,1)$.