

MATH 1231 Mathematics 1B Algebra S2 2008
Test 2 Version 2A (Green)

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1. (a) Put in matrix form and row reduce.

$$\begin{pmatrix} 1 & -1 & 2 \\ 2 & -1 & 1 \\ 3 & 1 & -6 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -1 & 2 \\ 0 & 1 & -3 \\ 0 & 0 & 0 \end{pmatrix}$$

Thus we can let $\alpha_3 = t$, hence $\alpha_2 = 3t$ and $\alpha_1 = -2t + 3t = t$.
Thus

$$\begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix} = t \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}, \quad t \in \mathbb{R}$$

- (b) Part (a) implies that $\mathbf{v}_3 = -3\mathbf{v}_2 - \mathbf{v}_1$ and so if T is to be linear it must satisfy $T(\mathbf{v}_3) = -3T(\mathbf{v}_2) - T(\mathbf{v}_1)$. A quick calculation will show that T doesn't satisfy this and hence can not be linear.

2. A quick calculation will show that

$$1 \times T \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + 2 \times T \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} - 5 \times T \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \neq T \begin{pmatrix} 1 \\ 2 \\ -5 \end{pmatrix}$$

- 3.

$$\left(\begin{array}{cccc|c} 1 & -1 & 2 & 3 & -1 \\ 0 & 1 & 3 & -1 & 4 \\ 1 & 0 & 7 & 3 & 6 \\ -2 & 5 & 5 & -9 & 6 \end{array} \right) \rightarrow \left(\begin{array}{cccc|c} 1 & -1 & 2 & 3 & -1 \\ 0 & 1 & 3 & -1 & 4 \\ 0 & 0 & 2 & 1 & 3 \\ 0 & 0 & 0 & 0 & -8 \end{array} \right)$$

thus $\text{rank}(A) = \text{no. of leading columns} = 3$ and $\text{nullity}(A) = \text{no. of non-leading columns} = 1$.

We can also see that \mathbf{b} is not in the image of A due to the last column in the augmented matrix being a leading column.

4. From what we learnt

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = A \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{4t} + B \begin{pmatrix} -1 \\ 3 \end{pmatrix} e^{9t}$$

and so $y_1 = Ae^{4t} - Be^{9t}$ and $y_2 = 2Ae^{4t} + 3Be^{9t}$.