

MATH 1251 Mathematics 1B Algebra S2 2008
Test 2 Version 2A (Green)

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1. Page 68 of your notes. You lost marks if you used anything but the axioms.
2. (a) The dimension of $M_{2,2}$ is 4. Thus 3 vectors can not possibly span it.
(b) Stick the matrices as columns of a matrix (in any consistent way) and row reduce. You should get all columns to be leading. Hence, the set is linearly independent.
3. Let $\mathbf{x}, \mathbf{y} \in \mathbb{R}^3$. Then

$$\begin{aligned} T(\mathbf{x} + \mathbf{y}) &= \begin{pmatrix} 2(x_1 + y_1) - 7(x_2 + y_2) \\ 3(x_1 + y_1) + (x_2 + y_2) + (x_3 + y_3) \end{pmatrix} \\ &= \begin{pmatrix} 2x_1 - 7x_2 \\ 3x_1 + x_2 + x_3 \end{pmatrix} + \begin{pmatrix} 2y_1 - 7y_2 \\ 3y_1 + y_2 + y_3 \end{pmatrix} \\ T(\lambda\mathbf{x}) &= \begin{pmatrix} 2\lambda x_1 - 7\lambda x_2 \\ 3\lambda x_1 + \lambda x_2 + \lambda x_3 \end{pmatrix} \\ &= \begin{pmatrix} \lambda(2x_1 - 7x_2) \\ \lambda(3x_1 + x_2 + x_3) \end{pmatrix} \\ &= \lambda T(\mathbf{x}) \end{aligned}$$

$$A_T = \begin{pmatrix} 2 & -7 & 0 \\ 3 & 1 & 1 \end{pmatrix}$$

4. $3 \begin{pmatrix} 5 \\ -2 \end{pmatrix} + 4 \begin{pmatrix} 2 \\ 7 \end{pmatrix} = \begin{pmatrix} 23 \\ 22 \end{pmatrix}$