# MATH 1231 Mathematics 1B Algebra S2 2008 Version 4B (Pink)

Boris Lerner

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1. People didn't set this question out too well. You have to be careful with your setting out.

   First of all $\mathbf{0} \in S$ since $2 \times 0 + 4 \times 0 + 10 \times 0 = 0$.

   Now lets show closure under vector addition. Suppose

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}, \mathbf{y} = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} \in S.$$

   We need to show that $\mathbf{x} + \mathbf{y} \in S$. Note that

$$\mathbf{x} + \mathbf{y} = \begin{pmatrix} x_1 + y_1 \\ x_2 + y_2 \\ x_3 + y_3 \end{pmatrix}$$

   by definition of how to add vectors. Now, (watch how I set my work out carefully)

$$\begin{aligned} 2(x_1 + y_1) + 4(x_2 + y_2) + 10(x_3 + y_3) &= (2x_1 + 4x_2 + 10x_3) + (2y_1 + 4y_2 + 10y_3) \\ &= 0 + 0 \quad \text{since } \mathbf{x}, \mathbf{y} \in S \\ &= 0 \end{aligned}$$

   and so we can conclude that $\mathbf{x} + \mathbf{y} \in S$.

   Now lets show closure under scalar multiplication. Let $\lambda \in \mathbb{R}$ and $\mathbf{x}$ be as above. We need to show $\lambda\mathbf{x} \in S$. Note that

$$\lambda\mathbf{x} = \begin{pmatrix} \lambda x_1 \\ \lambda x_2 \\ \lambda x_3 \end{pmatrix}$$

by definition of how to multiply a vector by a scalar. Now,

$$\begin{aligned} 2(\lambda x_1) + 4(\lambda x_2) + 10(\lambda x_3) &= \lambda(2x_1 + 4x_2 + 10x_3) \\ &= \lambda \times 0 \quad \text{since } \mathbf{x} \in S \\ &= 0 \end{aligned}$$

and so we can conclude that $\lambda\mathbf{x} \in S$.

2. We need to determine whether we can find $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5 \in \mathbb{R}$ such that

$$\lambda_1 \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix} + \lambda_2 \begin{pmatrix} 2 \\ -2 \\ 4 \end{pmatrix} + \lambda_3 \begin{pmatrix} 0 \\ -1 \\ 3 \end{pmatrix} + \lambda_4 \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} + \lambda_5 \begin{pmatrix} 3 \\ -2 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 5 \end{pmatrix}.$$

We form the corresponding augmented matrix

$$\left(\begin{array}{rrrrr|r} -1 & 2 & 0 & 1 & 3 & 1 \\ 1 & -2 & -1 & 0 & -2 & -1 \\ -2 & 4 & 3 & -1 & 3 & 5 \end{array}\right)$$

and upon row reducing we get

$$\left(\begin{array}{rrrrr|r} -1 & 2 & 0 & 1 & 3 & 1 \\ 0 & 0 & -1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 \end{array}\right)$$

and since the last column is leading, we can see that there is no solution. Hence $\mathbf{b}$ is not in the column span of $A$.

3. (a) To find a linearly independent subset, we form the matrix with $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ as its columns

$$\begin{pmatrix} -1 & 3 & 4 \\ 3 & -8 & -5 \\ -1 & 2 & -3 \end{pmatrix}$$

After row reducing we get

$$\begin{pmatrix} -1 & 3 & 4 \\ 0 & 1 & 7 \\ 0 & 0 & 0 \end{pmatrix}$$

the only non-leading column in the third one which corresponds to $\mathbf{v}_3$. Hence, Span $S$= Span$\{\mathbf{v}_1, \mathbf{v}_2\}$.

(b) From the above, we can see that $\mathbf{v}_3$ is in the span of $\{\mathbf{v}_1, \mathbf{v}_2\}$. We need to find numbers $\lambda_1, \lambda_2 \in \mathbb{R}$ such that

$$\lambda_1 \begin{pmatrix} -1 \\ 3 \\ -1 \end{pmatrix} + \lambda_2 \begin{pmatrix} 3 \\ -8 \\ 2 \end{pmatrix} = \begin{pmatrix} 4 \\ -5 \\ -3 \end{pmatrix}$$

We form the corresponding augmented matrix (note that is identical to the one before)

$$\left(\begin{array}{rr|r} -1 & 3 & 4 \\ 3 & -8 & -5 \\ -1 & 2 & -3 \end{array}\right)$$

and upon row reducing we get

$$\left(\begin{array}{rr|r} -1 & 3 & 4 \\ 0 & 1 & 7 \\ 0 & 0 & 0 \end{array}\right)$$

Back substituting we get $\lambda_2 = 7$ and $\lambda_1 = 17$ and so

$$17\mathbf{v}_1 + 7\mathbf{v}_2 = \mathbf{v}_3$$