MATH 1251 Mathematics 1B Algebra S2 2008 Version 3B (Pink)

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1.

$$\begin{aligned}
&\Rightarrow \text{Im}\left(\frac{4+3i}{a+2i}\right) = 10\\
&\Rightarrow \text{Im}\left(\frac{(4+3i)(a-2i)}{a^2+4}\right) = 10\\
&\Rightarrow 3a - 8 = 10a^2 + 40\\
&\Rightarrow 5a^2 - 15a + 24 = 0
\end{aligned}$$

which (unfortunately??) has no solutions.

2. We need to solve $z^6 = -64i = 64e^{2k\pi - \frac{\pi}{2}}$. Let $z = re^{i\theta}$ and so $r^6 = 64$ and so $r = 2$. Also, $6\theta = 2k\pi - \frac{\pi}{2}$ and so $\theta = \frac{4k\pi - \pi}{12}$. Thus

$$z = 2e^{\frac{4k\pi-\pi}{12}} \quad , k = -2, -1, 0, 1, 2, 3.$$

So $z = 2e^{-\frac{3i\pi}{4}}, 2e^{-\frac{5i\pi}{12}}, 2e^{-\frac{i\pi}{12}}, 2e^{\frac{i\pi}{4}}, 2e^{\frac{7i\pi}{12}}, e^{\frac{11i\pi}{12}}$.

3. This was the hard question of the test. Note that Daniel Chan did go through a similar one in lectures. We have:

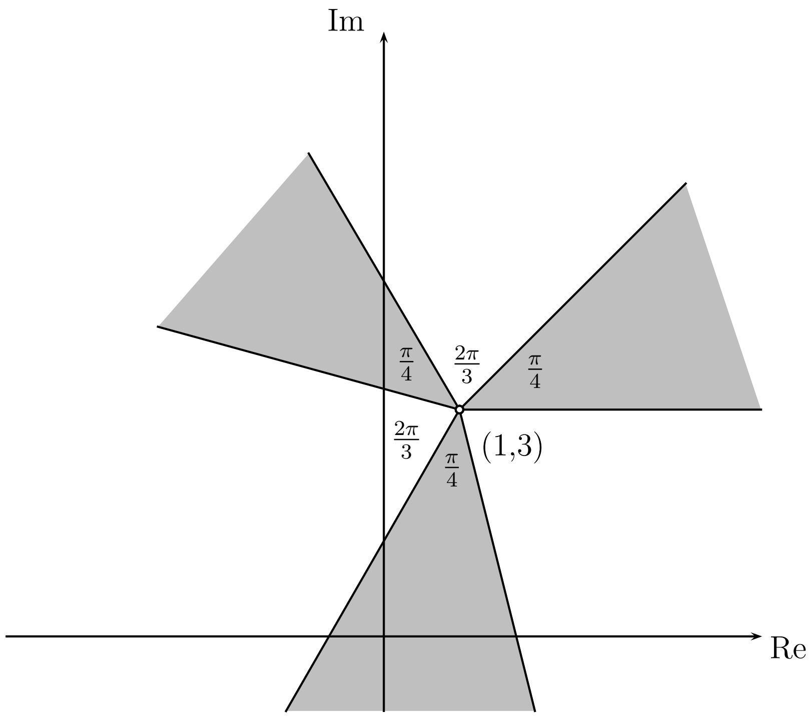
$$0 \leq \text{Arg}\left((z-1-3i)^3\right) \leq \frac{3\pi}{4}$$

This implies

$$\begin{aligned}
&0 \leq 3\text{Arg}\,(z-1-3i) + 2k\pi \leq \frac{3\pi}{4}\\
\Rightarrow \quad &-\frac{2k\pi}{3} \leq \text{Arg}\,(z-1-3i) \leq \frac{3\pi - 8k\pi}{12}
\end{aligned}$$

For $k = -1, 0, 1$ this gives

$$0 \leq \operatorname{Arg}(z - 1 - 3i) \leq \frac{\pi}{4}$$
$$-\frac{2\pi}{3} \leq \operatorname{Arg}(z - 1 - 3i) \leq \frac{-5\pi}{12}$$
$$\frac{2\pi}{3} \leq \operatorname{Arg}(z - 1 - 3i) \leq \frac{11\pi}{12}$$



4. (a) We did a very similar question in our tute. It's a GP with common ratio of $e^{2i\theta}$ and there are n terms. Thus the sum is

$$\frac{e^{i\theta}\left(1 - e^{2in\theta}\right)}{1 - e^{2i\theta}} = \frac{e^{i\theta}e^{in\theta}\left(e^{-in\theta} - e^{in\theta}\right)}{e^{i\theta}\left(e^{-i\theta} - e^{i\theta}\right)}$$
$$= \frac{-e^{in\theta}2i\sin n\theta}{-2i\sin\theta}$$
$$= \frac{e^{in\theta}\sin n\theta}{\sin\theta}$$

(b) Taking the imaginary part of part (i) we get $\frac{\sin^2 n\theta}{\sin\theta}$.