MATH 1251 Mathematics 1B Algebra S2 2008 Version 2B (Pink)

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1. We need

$$\begin{aligned} \text{Re}\left(\frac{1+2i}{a+3i}\right) &= 1 \\ \Rightarrow \quad \text{Re}\left(\frac{(1+2i)(a-3i)}{a^2+9}\right) &= 1 \\ \Rightarrow \quad a^2-a+3 &= 0 \end{aligned}$$

   which (unfortunately??) has no real solutions.

2.

$$\begin{aligned} \left(\sqrt{3}-i\right)^{50} &= \left(2e^{-\frac{i\pi}{6}}\right)^{50} \\ &= 2^{50}e^{-\frac{50i\pi}{6}} \\ &= 2^{50}e^{-\frac{25i\pi}{3}+8i\pi} \\ &= 2^{50}e^{\frac{-i\pi}{3}}. \end{aligned}$$

   The real part is $2^{50}\cos -\frac{\pi}{3} = 2^{49}$ and the imaginary part is $2^{50}\sin -\frac{\pi}{3} = -2^{49}\sqrt{3}$.

3. This was the hard question of the test. Note that Daniel Chan did go through a similar one in lectures. We have:

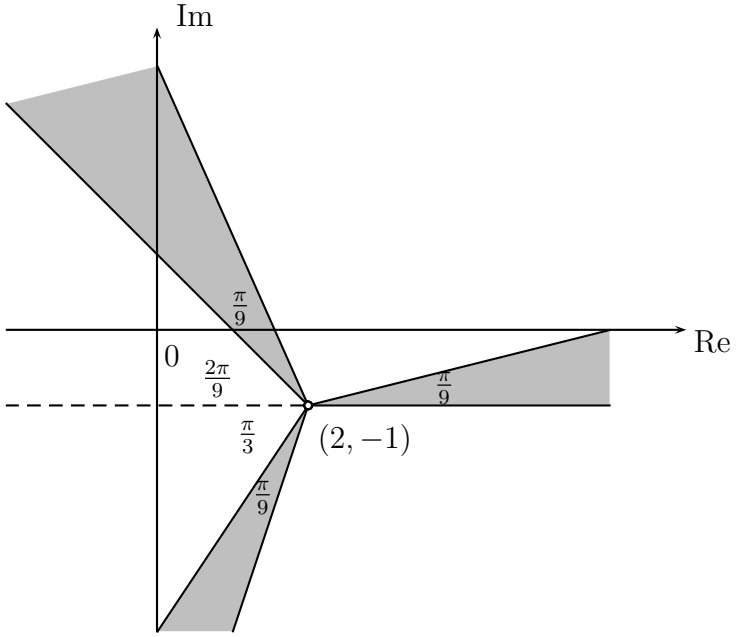
$$0 \leq \text{Arg}\left((z-2+i)^3\right) \leq \frac{\pi}{3}$$

   This implies

$$0 \leq 3\text{Arg}\,(z-2+i) + 2k\pi \leq \frac{\pi}{3}$$

$$\Rightarrow \quad -\frac{2k\pi}{3} \leq \operatorname{Arg}(z-2+i) \leq \frac{\pi-6k\pi}{9}$$

For $k=-1,0,1$ this gives $0 \leq \operatorname{Arg}(z-2+i) \leq \frac{\pi}{9}$, $-\frac{2\pi}{3} \leq \operatorname{Arg}(z-2+i) \leq -\frac{5\pi}{9}$, $\frac{2\pi}{3} \leq \operatorname{Arg}(z-2+i) \leq \frac{7\pi}{9}$.



4. We first solve $z^7 = -1 = e^{(\pi+2k\pi)i}$ and so

$$z = e^{\frac{i\pi(2k+1)}{7}}, \quad k = -3, \ldots, 3$$

With this, we factorise z^7+1 first into complex linear factors, but groups the conjugates together:

$$\begin{aligned} z^7+1 &= (z+1)\left[\left(z-e^{\frac{-5\pi i}{7}}\right)\left(z-e^{\frac{5\pi i}{7}}\right)\right]\left[\left(z-e^{\frac{-3\pi i}{7}}\right)\left(z-e^{\frac{3\pi i}{7}}\right)\right]\left[\left(z-e^{\frac{-\pi i}{7}}\right)\left(z-e^{\frac{\pi i}{7}}\right)\right] \\ &= (z+1)\left(z^2-2\cos\frac{5\pi}{7}+1\right)\left(z^2-2\cos\frac{3\pi}{7}+1\right)\left(z^2-2\cos\frac{\pi}{7}+1\right) \\ &= (z+1)\left(z^2+2\cos\frac{2\pi}{7}+1\right)\left(z^2-2\cos\frac{3\pi}{7}+1\right)\left(z^2-2\cos\frac{\pi}{7}+1\right) \end{aligned}$$

I didn't deduct marks if you stopped at the second last line.