Mixing/Diffusion

- Mixing via molecular diffusion is very weak compared to turbulent mixing.
- Turbulence caused by unstable conditions (i.e. dense water over lite water – convection) or shear in the flow (which may be caused by wind/waves).

Ocean Mixing by Flux Gradient Law

Can use the molecular diffusion formula to approximate turbulent mixing (but the value of $K$ needs to be much bigger).

$$\Gamma = -K \frac{\partial C}{\partial x}$$

- Flux (m$^2$/s)
- Vertical or horizontal diffusivity (m$^2$/s)
- Concentration gradient (could be salinity, temperature, oxygen etc)

Diffusion effects will therefore be strongest in regions with high concentration gradients or biggest $K$. 

Molecular diffusion is much slower than turbulent diffusion

- Clear fluid + red dye = pink water

Flux of $C$

Diffusivity

depends on temperature and viscosity (these affect the speed of particles)

Gradient of $C$
Molecular versus Turbulent (eddy) Diffusivity

Eddy (turbulent) diffusivities are usually many orders of magnitude larger than their molecular counterparts.

Diffusion in water (molecular) \( \sim 10^{-11} - 10^{-7} \text{ m}^2/\sec \)

Vertical diffusion in the ocean, \( \kappa_z \sim 10^{-5} - 10^{-3} \text{ m}^2/\sec \)

Horizontal diffusion in the ocean, \( \kappa_x \sim 1 - 1000 \text{ m}^2/\sec \)

- Deep ocean mixing occurs near rough topography
- Surface layer turbulence can be caused by wind mixing

More appropriate to consider mixing along and across isopycnals

Richardson No:

Predicts the onset of turbulence

\[
Ri = \frac{N^2}{\left(\frac{du}{dz}\right)^2}
\]

Where \( N \) is the Buoyancy Freq, \( u \) is a horizontal current speed. If \( Ri < 0 \) then density variations enhance turbulence (i.e. \( N^2 < 0 \)) i.e water column is unstable – turbulent convective mixing

If \( Ri > 0 \) (i.e. stable stratification) then velocity shear \( (du/dz) \) must be large to generate turbulence.

If \( Ri > \frac{1}{4} \) no turbulence is found

If \( Ri < \frac{1}{4} \) turbulence may be found

If \( Ri < 0 \) always turbulent (as the water column is unstable)

Determine if turbulence will occur in the following examples:

What is \( du/dz \)?

Upper Ocean

Deep Ocean

\[
Ri = \frac{(2\pi/(10x60))^2}{(0.5/10)^2} = 0.04 < \frac{1}{4} \text{ (TURBULENT)}
\]

\[
Ri = \frac{(2\pi/(7x60x60))^2}{(0.1/1000)^2} = 5.8 > \frac{1}{4} \text{ (STABLE)}
\]

Total Heat and Salt Fluxes

\[
V_T = u \times L \times H \quad \text{m}^3/\text{s}
\]

\[
Q_T = \rho c_p u T \times L \times H \quad \text{J/s}
\]

\[
S_{(H)} = \rho u S \times L \times H \quad \text{Kg/s}
\]
Volume transport = $uHL$

$= 0.05 \text{m/s} \times 1000 \text{m} \times 2800 \text{km}$

$= 140 \times 10^6 \text{m}^3/\text{s} = 140 \text{Sv}$

Heat Flux $Q = \rho c_p u T \times HL$

$= 1028 \text{kg/m}^3 \times 4000 \text{J/kg}^\circ \text{C} \times 0.05 \text{m/s} \times 6 \text{C} \times 1000 \text{m} \times 2800 \text{km}$

$= 3.454 \times 10^{15} \text{J/S (W)}$

Salt Flux $S = \rho u S \times HL$

$= 1028 \text{kg/m}^3 \times 0.05 \text{m/s} \times 34 \text{kg/1000kg} \times 1000 \text{m} \times 2800 \text{km}$

$= 4.893 \times 10^9 \text{kg/s}$

**Newton's Laws of Motion**

An object will continue to move in a straight line and at a constant speed unless acted on by a NET force.

The change in the velocity (speed and/or direction) of an object (e.g. a bit of water) is proportional to the force and inversely proportional to the mass of the object.

$\mathbf{F} = \mathbf{ma}$

$\mathbf{a} = \frac{1}{m} \Sigma \mathbf{F}$ but $m = \rho V$

so, $\mathbf{a} = \frac{1}{\rho V} \Sigma \mathbf{F}$

Oceanographers will consider force per unit volume.

Acceleration $a = \frac{du}{dt}$

So, $\frac{du}{dt} = \frac{1}{\rho} \Sigma F_u$

Friction – predominantly at boundaries*  
Wind – only at surface boundary  
Seismic Forces – Occasional impulse only  
Coriolis* - due to the earths rotation  
Buoyancy – due to vertical T,S differences (related to heat and water fluxes at the ocean surface)

* These forces are only experienced by fluid in motion.

Ocean Dynamics

What are the forces that drive the Ocean?

- Gravity (earth, sun and moon)
- Pressure force
- Friction – predominantly at boundaries*
- Wind – only at surface boundary
- Seismic Forces – Occasional impulse only
- Coriolis* - due to the earths rotation
- Buoyancy – due to vertical T,S differences (related to heat and water fluxes at the ocean surface)

u,x: west to east  v,y: south to north  w,z: up to down
Forces on a Parcel of Water

- Gravity
- Coriolis
- Pressure
- Friction

\[ \frac{du}{dt} = \frac{1}{\rho} \left( F_g + F_C + F_P + F_f + \ldots \right) \]

If we know the acceleration of the water at different times, we can also figure out its velocity at different times and the position of a water particle at different times.

\[ \frac{du}{dt} \rightarrow u \rightarrow x \]

e.g., water starting at rest and at \( x=0 \), accelerates at \( 0.1 \text{m/s}^2 \) for 10 seconds and then has zero acceleration for 10 seconds.

Mathematically this process is integration.

Newton's Laws of Motion

What are the forces acting in the vertical direction?

- The boxes weight is acting downwards (mg)
- The pressure at the top of the box is also trying to force the box downwards.
- But the pressure at the bottom of the box is trying to force it upwards (the difference in the pressure forces is just the buoyancy force discussed for Archimedes)

\[ \frac{dw}{dt} = g \]

But acceleration in the z direction is just: \( \frac{dw}{dt} \), so

\[ \frac{dw}{dt} = - \frac{1}{\rho} \frac{dp}{dz} + g \]
Newton's Laws of Motion

Vertical direction

\[
\frac{dw}{dt} = \frac{1}{\rho} \frac{dp}{dz} + g
\]

Vertical acceleration

Buoyancy force

Weight

We can normally make this even simpler.

We can use SCALE ANALYSIS to evaluate the size of each term, compare their magnitudes, and neglect the small terms.

e.g. Suppose \( A = B + C \)

If we know that \( B=0.00001 \) and \( C=10 \), then to a good approximation we could simplify this equation to \( A = C \)

e.g. Suppose \( D = E + F \)

If \( D=0.00001 \) and \( E=10 \), then to a good approximation we could simplify this equation to \( E = -F \)

Exercise

• Note that the Equatorial Pacific is ~ 10,000 km wide and the upwelling is concentrated in a 50 km wide band at the equator. Once the winds start blowing it takes about a day for the upwelling to spin up

\[
\frac{dw}{dt} = \frac{1}{\rho} \frac{dp}{dz} + g
\]

• What is \( \frac{dw}{dt} \)?
• How does it compare in size to gravity?

Upwelling = 100,000,000m^3/s. This is coming up through an area of: \( A=10,000,000 \times 50,000 = 5 \times 10^{11} \text{m}^2 \). So (remember volume transport):

\[
U= \frac{(100,000,000 \text{m}^3/\text{s})(5 \times 10^{11} \text{m}^2)}{1 \times 24 \times 60 \times 60} = 0.0002 \text{m/s} \text{ (very small!)}
\]

\[
U= (100,000,000 \text{m}^3/\text{s})(5 \times 10^{11} \text{m}^2) = 0.0002 \text{m/s} \text{ (very small!)}
\]

If the wind were to stop and then start again it would take about a day to start upwelling at that rate again i.e. It takes 1 day to reach a velocity of 0.0002 m/s.

So we can approximate our acceleration to

\[
\frac{dw}{dt} = W \frac{T}{1 \times 24 \times 60 \times 60} = 2 \times 10^{-3} \text{ms}^{-2} << \text{g = 10ms}^{-2}
\]

In general over the ocean vertical acceleration is much smaller than \( g \). This means that in the vertical equation \( g \) and \( \frac{1}{\rho} \frac{dp}{dz} \) must be of similar magnitude.
This (\(dw/dt\)) is much smaller than gravity, hence, gravity cannot be balanced by \(dw/dt\). Generally most ocean circulation of period > 1 day satisfies the hydrostatic balance.

\[
g \frac{dz}{dp} = \frac{\rho g}{\Delta \rho} = \frac{1 \times 10^{-4}}{3 \times 10^4} = 3 \times 10^{-11} \text{ m sec}^{-2}
\]

This is called the hydrostatic equation.

We can integrate this equation since density and \(g\) are essentially constant.

\[
\int_0^h \frac{dp}{dz} \, dz = \int_0^h \rho g \, dz
\]

Or simply

\[
p = \int_0^h \rho g \, dz = \int_0^h \rho g \, dz
\]

Which you are hopefully familiar with already!
Newton's Laws of Motion

Horizontal direction

Just like vertical pressure gradients can create a force on the water, so can horizontal gradients.

\[
\begin{align*}
\Delta x & \quad \Delta y & \quad \Delta z \\
\rho & \quad \Delta V & \\
P & \quad P + \Delta P & \\
\end{align*}
\]

We can now forget the vertical – we’ve done that already. What about forces in the horizontal?

Barotropic and Baroclinic Motion

Remember, \( p = \rho g z \)

\[
\begin{align*}
\rho_0 & = 1027 \\
\end{align*}
\]

\[
\begin{align*}

\text{Surface: } & \Delta A = \Delta y \Delta z \\
\text{Volume: } & \Delta V = \Delta x \Delta y \Delta z \\
\text{Mass: } & m = \rho \Delta V = \rho \Delta x \Delta y \Delta z \\
\end{align*}
\]

Barotropic and Baroclinic Motion

Remember, \( p = \rho g z \)

\[
\begin{align*}
p_1 & = 1027 \times 10^5 \text{ Pa} = 1027 \times 100 \text{ kg/m/s}^2 \\
p_2 & = 410.8 \times 10^5 \text{ Pa} \\
p_3 & = 616.2 \times 10^5 \text{ Pa} \\
\end{align*}
\]

\[
\begin{align*}
\rho_1 & = 1027 \\
\rho_2 & = 410.8 \\
\rho_3 & = 616.2 \\
\end{align*}
\]

\[
\begin{align*}
\frac{du}{dt} = \frac{1}{\rho} \frac{\partial p}{\partial x} = \frac{1}{\rho} \frac{1}{\rho} \frac{\Delta p}{\Delta x} \\
\frac{du}{dt} = \frac{1}{1027} \cdot \frac{205.4 - 102.7}{0.05} = -2 \text{ m/s}^2 \\
\frac{du}{dt} = \frac{1}{410.8} \cdot \frac{410.8 - 308.1}{0.05} = -2 \text{ m/s}^2 \\
\frac{du}{dt} = \frac{1}{616.2} \cdot \frac{616.2 - 513.5}{0.05} = -2 \text{ m/s}^2 \\
\end{align*}
\]
Barotropic and Baroclinic Motion

Remember, \( p = \rho g z \), and

\[
\frac{du}{dt} = -\frac{1}{\rho} \frac{\partial p}{\partial x}
\]

\( \rho_1 = 1026 \) km \(-3\)

\( \rho_2 = 1028 \) km \(-3\)

Barotropic velocity is constant with depth

Baroclinic velocity changes with depth

Motion due to surface slopes

Motion due to density differences

Barotropic Currents move the whole water column. Can be induced by horizontal changes in elevation, which produce a pressure gradient throughout the whole water column.

Baroclinic Currents vary over depth. This can be induced by a horizontal density gradient. This effect is known as the thermal wind balance (more later).
Barotropic and Baroclinic Motion

Remember, \( p = \rho g z \), and \( \frac{du}{dt} = \frac{1}{\rho} \frac{\partial \rho}{\partial x} \)

\( \rho_1 < \rho_2 \)

Motion due to density differences

Barotropic:
Isobaric surfaces are parallel to isopycnic surfaces

Baroclinic:
Isobaric and isopycnic surfaces are NOT parallel

Where are we likely to find barotropic conditions in the ocean?
- Well-mixed surface layers
- Shallow shelf seas (particularly where shelf waters are well mixed by tidal currents)
- Deep ocean (below the permanent thermocline)

Where are we likely to find baroclinic conditions in the ocean?
- Regions of fast surface currents

For a constant density ocean, we can write the pressure gradient in an easier way.
Remember, \( p = \rho g z \), and \( \frac{du}{dt} = \frac{1}{\rho} \frac{\partial \rho}{\partial x} \)
Barotropic Ocean

For a constant density ocean, we can write the pressure gradient in an easier way.

Remember, \( p = \rho g \), and

\[
\frac{du}{dt} = -\frac{1}{\rho} \frac{\partial \rho}{\partial x} \Delta p
\]

\[
\frac{\partial p}{\partial x} = \frac{\Delta p}{\Delta x} = \frac{p_2 - p_1}{\Delta x}
\]

\[
\frac{\partial \rho}{\partial x} = \frac{\Delta \rho}{\Delta x} = \frac{\rho_2 - \rho_1}{\Delta x}
\]

So we are left with

\[
\frac{du}{dt} = -g \frac{\partial \eta}{\partial x}
\]

For a constant density ocean, we can write the pressure gradient in an easier way.

Remember, \( p = \rho g \), and

\[
\frac{du}{dt} = -\frac{1}{\rho} \frac{\partial \rho}{\partial x} \Delta p
\]

\[
\frac{\partial p}{\partial x} = \frac{\Delta p}{\Delta x} = \frac{p_2 - p_1}{\Delta x}
\]

\[
\frac{\partial \rho}{\partial x} = \frac{\Delta \rho}{\Delta x} = \frac{\rho_2 - \rho_1}{\Delta x}
\]

So we are left with

\[
\frac{du}{dt} = -g \frac{\partial \eta}{\partial x}
\]

Barotropic Motion

For a constant density ocean, the acceleration of water just depends on what is happening at the surface i.e. the slope of the sea surface!

\[
\frac{du}{dt} = -g \frac{\partial \eta}{\partial x}
\]

\[
\frac{\Delta \eta}{\Delta x} = \frac{(d + \eta_1 + \eta_2)/2 - (d + \eta_1)/2}{\Delta x} = \frac{\Delta \eta_2}{\Delta x}
\]

\[
\frac{du}{dt} = -10 \left( \frac{\Delta \eta}{\Delta x} \right) = -2 \text{ms}^{-2}
\]

Pressure Gradients

• Water tends to move from high to low pressure (consider puncturing a car tyre).

• Horizontal pressure differences are evident in gradual gradients in sea level over the ocean.
Summary:
• Start with Newton’s Law: the acceleration of a parcel of water is related to the sum of forces acting on that water parcel.
  • We can break down the acceleration and forced into those acting in the x, y and z directions
  • In the z-direction, vertical velocities/accelerations are small, which leaves us with the hydrostatic equation
    \[ \frac{dp}{dz} = \rho g \]
    This just means that the weight of water is balanced by the vertical pressure gradient (in other word the buoyancy force)
  • In the x-direction: \[ \frac{du}{dt} = -\frac{1}{\rho} \frac{\partial p}{\partial x} \]
    So the water acceleration is driven by horizontal differences in pressure (these differences may be related to horizontal differences in surface height (barotropic) or in density (baroclinic))
  • For a barotropic (constant density) situation: \[ \frac{du}{dt} = -g \frac{\partial \eta}{\partial x} \]
  • Similarly in the y-direction: \[ \frac{dv}{dt} = -\frac{1}{\rho} \frac{\partial p}{\partial y} , \text{ or } \frac{dv}{dt} = -g \frac{\partial \eta}{\partial y} \]
  • But there are other forces out there …

The Coriolis Force

\[ \ddot{a} = \frac{1}{m} \sum F \]

\[ \frac{du}{dt} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \text{Coriolis force} + \ldots \]

\[ \frac{dv}{dt} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \text{Coriolis force} + \ldots \]

\[ \frac{d\rho}{dz} = \rho g \]

Object moving frictionlessly over the surface of a very shallow parabolic dish. The object has been released in such a way that it follows an ellipse-shaped trajectory.

Left: the motion as observed from the inertial point of view. The gravitational force pulling the object toward the bottom (center) of the dish is proportional to the distance of the object from the center. This force causes the elliptical motion.

Right: the motion as seen from a co-rotating point of view. In this frame, the inward gravitational force is balanced by the outward centrifugal force. The only unbalanced force is Coriolis, and the motion is an inertial circle.
The Coriolis Force

The strength of the Coriolis force varies with latitude. It is proportional to the Coriolis parameter $f = 2\Omega \sin(\Phi)$, where $\Phi$ is latitude, and $\Omega$ is the angular velocity (in radians per second). It is maximum at the poles, zero at the equator, and changes sign from NH to SH.

The acceleration due to the Coriolis force is:
- $fv$ in the x-direction (east-west), and
- $-fu$ in the y-direction (north-south)

The Coriolis Force Summary

- Acceleration due to the Coriolis force is $fv$ (x direction) and $-fu$ (y direction)
- acts only if water/air is moving
- acts at right-angles to the direction of motion
- causes water/air to move to the right in the northern hemisphere
- causes water/air to move to the left in the southern hemisphere

The Equations of motion

Horizontal Equations:

$\frac{du}{dt} = -\frac{1}{\rho_v} \frac{dp}{dx} + fv$

$\frac{dv}{dt} = -\frac{1}{\rho_v} \frac{dp}{dy} - fu$

Vertical Equation:

Pressure Gradient force = Gravitational Force

$\frac{dp}{dz} = \rho g$

For a Barotropic Ocean we can write the horizontal equation in a way that’s easier to understand.
Coriolis Force

- What is the value of $\Omega$ (the earth rotates 360° every day)?
- What is $f$ at the north pole, the equator, 30N and 30S?
- What is the acceleration of water flowing at 1m/s at 30S? What pathway will the water trace out?

$\Omega$ – earth rotates $2\pi$ radians in 1 day: $2* \pi / (24*60*60) = 7.2722 \times 10^{-5}$ s$^{-1}$

$f=2\Omega \sin (\text{latitude})$

At the north pole latitude =90°, $\sin 90=1$, $f=2\Omega$
At 30N $\sin(30)=0.5$, $f= \Omega$, At 30S $\sin(-30)=-0.5$, $f= -\Omega$

Say motion is in west to east direction, i.e. $u=1$ m/s, $v=0$ m/s, $\frac{du}{dt}=f$, $\frac{dv}{dt}=0$, $\frac{dv}{dt}=fv= -\Omega$ (1) = $7.2 \times 10^{-5}$ m/s$^2$

SO Acceleration is in south to north direction (what would it be if the initial motion was north to south?). If no other forces acting the water would flow around in a circle with frequency $|f|$.

This is called inertial motion

If the only force acting on a body is Coriolis, Newton’s equations would be:

$$\frac{du}{dt} = fv$$
$$\frac{dv}{dt} = -fu$$

What does this mean?

Foucault’s Pendulum - simple example of Coriolis

If the only force acting on a body is Coriolis, Newton’s equations would be:

$$\frac{du}{dt} = fv$$
$$\frac{dv}{dt} = -fu$$

What does this mean?
If the only force acting on a body is Coriolis, Newton’s equations would be:

\[
\frac{du}{dt} = fv
\]

\[
\frac{dv}{dt} = -fu
\]

What does this mean?

Scaling arguments:
These are the equations you need when there are both pressure and Coriolis forces in play. But this is not always going to be the case. If there are no surface slopes or horizontal density differences then there will be no pressure force (i.e. left with \( du/dt=fv \) etc.)

What forces are important in a bath tub?
What kind of speeds will the water get up to? What kind of accelerations? What surface slopes?

The Equations of Motion

**Horizontal Equations:**
Acceleration = Pressure Gradient Force + Coriolis

\[
\frac{du}{dt} = \frac{1}{\rho} \frac{\partial p}{\partial x} + fv
\]

\[
\frac{dv}{dt} = \frac{1}{\rho} \frac{\partial p}{\partial y} - fu
\]

Or, for a Barotropic Ocean:

\[
\frac{du}{dt} = \frac{g}{\rho} \frac{\partial}{\partial x} \eta
\]

\[
\frac{dv}{dt} = \frac{g}{\rho} \frac{\partial}{\partial y} \eta - fu
\]

**Vertical Equation:**
Pressure Gradient Force = Gravitational Force

\[
\frac{dp}{dz} = \rho g
\]

Scaling arguments:
What forces are important in a bath tub?
What kind of speeds will the water get up to? What kind of accelerations? What surface slopes?

Size of the pressure force:

\[
\frac{d\eta}{dx} = 0.1 \text{ m/m} = 0.1
\]

\[
\frac{d\eta}{dx} = 10 \times 0.1 = 1 \text{ m/s}^2
\]

Size of the Coriolis force:

\[
f_u = 7 \times 10^{-5} \times 1 = 7 \times 10^{-5} \text{ m/s}^2
\]

So Coriolis<<Pressure, so we can neglect rotation effects

Acceleration in the bathtub is driven by pressure differences (due to changes in surface slopes)
We conduct a scaling analysis on our equations of motion.

To find further simplifications for motions with a period greater than ~ 10 days

Scaling Analysis:

\[
\begin{align*}
T \approx 10 \text{ days} &= 6.64 \times 10^5 \text{ s} \\
u, v &\sim U \sim 1 \text{ ms}^{-1} \\
f &\sim 10^{-4} \text{ s}^{-1}
\end{align*}
\]

3.4 Geostrophic Balance

In the equation of motion, acceleration is 1/100 smaller than the Coriolis Force and Pressure Force.

Thus the balance is between Pressure and Coriolis Forces - Geostrophic Balance.

Geostrophic Balance: How is it set up?

- What does the geostrophic balance mean physically?
- Suppose we have a difference in sea-level height.
- Water will want to move from the region of high pressure towards the region of low pressure.
Geostrophic Balance: How is it set up? (cont)

- As the water starts to move, the Coriolis effect (rotation) deflects the water to the right (NH) or left (SH).
- The water keeps getting deflected until the force due to the pressure difference balances the Coriolis force.
- This balance is called a **geostrophic balance** and the resulting current is referred to as a **geostrophic current**.

You are familiar with Pressure differences in the atmosphere...

Which direction is the Geostrophic wind? (f <0 SH)
Concept Problem (NH):
In the NH: Which way does the current flow if sea level height is increasing towards the South? West? North?

\[ fb = \frac{g}{\eta} \frac{d\eta}{dx} \]
\[ fu = -g \frac{d\eta}{dy} \]

Geostrophy Problem 2:
Which direction does the water flow around this pressure feature if it is in the Northern Hemisphere?

Geostrophy Problem 3:
Which direction does the water flow around this pressure feature if it is in the Southern Hemisphere?
Geostrophy Problems:

A certain ocean current has a height change of 1.1 m (increasing to the east) over its width of 100 km at 45° N. How fast is the current flowing?

\[ f = 1 \times 10^4 \text{ s}^{-1} \]
\[ g = 10 \text{ m s}^{-2} \]
\[ \Delta \eta = 1.1 \text{ m} \]
\[ \Delta x = 100 \times 1000 \text{ m} \]

\[ V = \text{? m/s} \]

Summary: Geostrophy is the balance between pressure forces and Coriolis. In most of the open ocean (away from boundaries) motion will become geostrophic after a few days. Geostrophy doesn’t work over short periods of time or small distances (other forces become dominant). Geostrophy also fails in regions where friction becomes important.

Recap

\[ \frac{du}{dt} = -\frac{1}{\rho} \frac{dp}{dx} + fv \]
\[ \frac{dv}{dt} = -\frac{1}{\rho} \frac{dp}{dy} - fu \]
\[ \frac{dp}{dz} = \rho g \]

Acceleration = Pressure + Coriolis Force

Weight of water is balanced by its buoyancy (or vertical pressure force)

After a period of time flow becomes steady, so, acceleration becomes zero. This occurs when the pressure force balances the Coriolis force.

\[ \frac{du}{dt} = \frac{1}{\rho} (F_g + F_C + F_p + F_f + ...) \]

The last force to consider is friction. This is only important at continental boundaries, at the bottom of the ocean, and at the surface (due to wind).

What will the friction term look like? We know that friction always tries to retard motion.

\[ \frac{du}{dt} = \text{?} \]