6 Atmosphere - Forecasting & Waves

Recent weather

Circulation & Vorticity
- \( C = \int \mathbf{u} \cdot ds \)
- \( \xi = \nabla \times \mathbf{u} \)
- \( \xi = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \)
- \( \xi = \xi_A + f \)

Vorticity is path independent

Change in vorticity:
- Due to horizontal divergence
- Due to tilting/twisting
- Due to temperature gradients

Scale Analysis
- \( U = O(10 \text{m s}^{-2}) \)
- \( L = O(10^6 \text{m}) \)
- \( f = O(10^{-4} \text{s}^{-1}) \)
- \( W = O(10^{-2} \text{m s}^{-1}) \)
- \( T = O(10^2 \text{K}) \)
- \( \Delta T = O(1 \text{K}) \)

Conservation of Vorticity
- \( \int \nabla \times \frac{\partial \mathbf{u}}{\partial x} \cdot \nabla \phi = \int (\xi + f) \frac{\partial \phi}{\partial x} \left( \frac{\partial \phi}{\partial y} \right) \)

In a barotropic atmosphere, this can be simplified to
- \( (\rho v - p) \frac{\partial (\xi + f)}{\partial x} = -\left( \frac{\partial f}{\partial y} \right) \frac{\partial \phi}{\partial x} \left( \frac{\partial \phi}{\partial y} \right) \)
- \( \left( \frac{\partial (\xi + f)}{\partial x} \right) = -\frac{\partial f}{\partial y} \frac{\partial \phi}{\partial x} \left( \frac{\partial \phi}{\partial y} \right) \)
- \( \left( \frac{\partial \phi}{\partial x} \right) = \frac{\partial (\xi + f)}{\partial x} \)
Circulation vs Vorticity

- Vorticity is path independent
- Vorticity brings back prognostic ability of equations
- Vorticity is conserved

\[ \frac{D}{Dt} \left( \frac{\xi + f}{p_2 - p_1} \right) = 0 \]

Vorticity - Forecasting

\[ f_b < f_a < 0 \]
\[ \xi_b \approx \xi_a \approx 0 \]

Hence,
\[ h_B < h_A \]

Subsidence, hence clear weather.

Example

- Determine what type of weather is most likely with the following synoptic situation based on conservation of vorticity. Assume \( \xi_A > \xi_B \)

Including the Upper Levels

Geopotential Height

\[ \Phi = \int g dz \]

Since \( \frac{\partial \Phi}{\partial z} = -\rho g \) and \( P = \rho RT \),

\[ \Phi = -\int RT d \ln P \]
Exercise:

- For the following vertical temperature profiles, what height, $z$, would you expect the 500 hPa surface to be at? Assume $R = 287 \text{ m}^2/\text{s}^2\text{K}$

Answers

- TRACE 1: 5533 m
- TRACE 2: 5262 m

Hence we define 1000-500 hPa thickness

The previous traces give a value of

- THICKNESS: 5379 m

Thickness small $\rightarrow$ cold
Thickness large $\rightarrow$ warm
System Tilt

• Since
  \[ \Phi = \frac{RTU}{\rho_d} \]
• Upper lows tend to overlay surface lows.
• If not, must be accumulation of cold air under upper low.
• Cold air normally to south and west. Why?
• Hence surface lows slope towards cold air.

Height variation

• Atmosphere is ~10 km thick, yet we always look at surface patterns …
• Are upper levels the same?

Re-enforcement

• Which situation is most likely to develop a low pressure system over SE Australia?

Steering

Rossby Waves

• Arise from conservation of potential vorticity with a varying coriolis parameter.
  \[ \frac{\partial \Phi}{\partial \lambda} = \frac{1}{f} \frac{1}{\beta} \frac{L_0}{T} \]
• Assume no divergence
  \[ \frac{D(\Phi + f)}{Dt} = 0 \]

What about divergence?

Low level convergence is balanced by upper level divergence - we say the surface system is ventilated. Between the surface and upper levels is a level where convergence/divergence is minimal - commonly taken to be around 500 hPa.
Rossby Waves

In one dimension...
\[ \omega = -\frac{\beta}{k} \]

Group velocity...
\[ \frac{\partial \theta}{\partial x} = \frac{\beta}{k^2} \]

Phase velocity...
\[ c = \frac{\omega}{k} \]

Total velocity...
\[ c_T = U - \frac{\beta}{k^2} \]

Assume an initial absolute vorticity of zero—i.e., a purely meridional displacement. Then
\[ \zeta + f_0 x = 0 \]  \hspace{1cm} (1)

Then
\[ \zeta_0 = f_0 - f_0 = -\beta y \]  \hspace{1cm} (2)

Considering only meridional displacements, then \( \zeta = \frac{\beta}{k} y \) and so
\[ \frac{\partial \theta}{\partial x} = -\frac{\beta}{k^2} y \]  \hspace{1cm} (3)

A solution is
\[ \frac{d\psi}{dx} = \sin(kx - ct) \]  \hspace{1cm} (4)

and substituting gives
\[ \frac{d^2\psi}{dx^2} = -\sin(kx - ct) \]  \hspace{1cm} (5)

and the phase speed of the wave, \( v \), is given by
\[ c = \frac{v}{k} \]  \hspace{1cm} (6)

Exercise:

• If there are 5 Rossby waves spanning the latitude 34S band, what is the velocity of the long wave trough over southern Australia?
• If the wave is embedded in a 35 m/s mean flow at 500 hPa and Adelaide is 1300 km west of Sydney, how long will it take the trough near Adelaide to reach Sydney?

Rossby Waves at 500 hPa
Steering cont…

- Short wave troughs (fronts, surface troughs, etc) move through long wave troughs.
- Can re-enforce each other.
- Is an extremely helpful way of predicting weather more than 4 days in advance.

16/07/07 00UTC

17/07/07 00UTC

18/07/07 00UTC

19/07/07 00UTC

20/07/07 00UTC

21/07/07 00UTC

22/07/07 00UTC

23/07/07 00UTC

24/07/07 00UTC

25/07/07 00UTC

Spaghetti charts

http://www.cdc.noaa.gov/map/images/enso/spag_sh_alltimes.html
Summary

• Writing the equations of motion in terms of vorticity allows us to regain some predictive capability.
• Considering the interplay between upper levels and lower levels, as well as horizontal temperature gradients, provides insight into weather development.
• The conservation of potential vorticity leads to the Rossby Wave - which can help us to determine how weather systems move.

Thermal Wind & Baroclinicity (Development)

Thermal Wind

Let’s just consider the usual parts:

\[ \frac{\partial \mathbf{v}}{\partial t} = -\mathbf{v} \times \nabla \times \mathbf{v} \]  

(6.17)

If we put \( \rho = \rho / RT \) and expand then we get,

\[ \frac{\partial \mathbf{v}}{\partial t} = \frac{-2 \mathbf{R} T}{\rho} \frac{\partial \mathbf{v}}{\partial y} - \frac{\mathbf{v} \cdot \nabla \mathbf{v}}{\rho} \]  

(6.18)

In strongly baroclinic areas the temperature gradients are much larger than the pressure gradients and we have

\[ \frac{\partial \mathbf{v}}{\partial t} \approx \frac{\mathbf{R} T}{\rho} \frac{\partial \mathbf{v}}{\partial y} \]  

(6.19)

We define the difference in the geostrophic wind between 2 heights as the thermal wind, \( w_f \). If we integrate between 2 pressure levels we can show that,

\[ w_f = \frac{R T}{\rho} \int_{\mathbf{P}_0}^{\mathbf{P}_1} \frac{d\mathbf{v}}{d\mathbf{z}} \]  

(6.20)

References

http://oceanworld.tamu.edu/resources/opw_tutorials/chapter10/chapter10.02.html
http://cimss.ssec.wisc.edu/goes/misc/99012.html

Thermal Wind

• Arises in a baroclinic atmosphere.
• Unlike geostrophic balance, does not just consider balance of CoF & PGF.
• But still a kind of A geostrophic balance...

Thermal Wind

• Geostrophic balance at each level to balance the different PGF.

\[ P_B \neq P_A - \int z \rho \chi dz \]

\[ P_B \neq \int z \rho \chi dz \neq P_A \]

\[ \rho B_{max} \neq \rho A_{max} \]

Hence

\[ \rho A < \rho B \]

\[ \frac{\partial \mathbf{v}}{\partial z} \neq 0 \]

Thermal wind
Thermal Wind - westerlies

Baroclinicity

The baroclinic term is:

\[ \alpha^2 J(\rho, p) \]

or

\[ \frac{R}{p} \nabla \cdot \left( \nabla \rho \times \nabla T \right) \]

Temperature variations induce a circulation

Baroclinic instability

- Is a method for transferring heat meridionally in a rotating environment.

Baroclinic Instability

Exercise:

- Where is development most likely on today’s chart?

From (3)