Globally...

- Mid-latitudes dominated by westerly flow, highs & lows, fronts etc. => Baroclinic.
- Tropics dominated by easterly trades and monsoons. Little pressure variation. => Barotropic.

Derivation of simplified vorticity equation

- Vorticity aids us in understanding development of systems.

\[ \frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla) \vec{v} = -\nabla \phi + \frac{1}{\rho} \nabla \times (\nabla \times \vec{v}) \]

\[ \nabla \times (\nabla \times \vec{v}) = 0 \]

First term:

\[ \frac{\partial \vec{v}}{\partial t} \]

Second term:

\[ \frac{1}{\rho} \nabla \times (\nabla \times \vec{v}) \]

where the first term is zero because \( \nabla \cdot \nabla \vec{v} = 0 \) always and the expansion of the second term into four terms is a vector identity.

Now the third term is zero because of the \( \nabla \cdot \nabla = 0 \) identity. The fourth term is zero if we assume horizontal winds.
In the equation we talked about last time, we allowed Winds to be vertical and thus retained a ‘tilting’ term.

Due to tilting/twisting

Due to Temperature gradients

Change in vorticity
With time

Due to Horizontal divergence

Due to tilting/ twisting

Due to Temperature gradients

Rossby Waves re-visited

- Rossby waves give us information about how surface systems are steered and re-enforced by the upper levels.

With gravity waves, we assumed \( f v \ll \frac{\partial \theta}{\partial z} \), to give

\[
\frac{du}{dt} = -\frac{\partial}{\partial z} \frac{dv}{dt} = -\frac{\partial}{\partial y} \frac{dv}{dt} = -h(\frac{du}{dx} - \frac{dv}{dy})
\]

which gave us the dispersion relation:

\[ \omega^2 = gh \tanh(kh) \]

For large \( \lambda, \kh \ll 1 \), then, \( \omega^2 = gh^2 k \) or \( c = \sqrt{gh} \).
From conservation laws:

\[ \frac{d}{dt} = 0 \]

\[ \frac{d}{dt} \left( \frac{\partial \Phi}{\partial y} \right) + \frac{\partial}{\partial x} \left( \frac{\partial \Phi}{\partial x} \right) = 0 \]

Equating gives the phase relation

**Rossby wave phase velocity** is westward, larger for longer wavelengths.

Propagation speeds of the order of 10 m/s: 4 waves around the hemisphere at 45.5 - 20 m/s (74 km/h) or around 2 days from Perth to Sydney.

\[ \frac{d}{dt} \left( \frac{\partial \Phi}{\partial y} \right) + \frac{\partial}{\partial x} \left( \frac{\partial \Phi}{\partial x} \right) = 0 \]

Assuming a solution of form,

\[ \Psi = \Psi_0 \exp\left[(kx + ly - \omega t)\right] \]

**First term:**

\[ \frac{d \Psi}{dt} = -ik\Psi \]

\[ \frac{d}{dt}(\nabla \Psi) = \omega (\epsilon^2 + \beta^2) \Psi \]

**Second term:**

Equating gives the phase relation

**Blocking**

- Long wavelength waves move only slowly eastwards,
  \[ \epsilon \Phi = \frac{U - \beta k}{k} \]

- If \( U = 25 \text{ m/s} \) (~90 km/h), wave will be stationary for \( k \approx 7800 \text{ km} \) at 45\(^\circ\), i.e. ~3 waves round hemisphere.

- In addition, waves can "split" at a latitude.
- Leads to blocking - pattern stops propagating - cool or hot air builds.
Synoptic pattern then re-enforces the upper level (for eg: by dragging more hot air southwards, increasing thickness. => re-enforcing the block (note the deepening of the long wave ridge over east Australia during the period).

• Such a situation was responsible for the Jan-Feb 2009 south Australian heatwave.
• Hottest temps on record in northern Tas.
• Hottest nights on record in S.A.
• Unprecedented southern extent.

Heat Wave climatology

Adelaide

Melbourne

500hPa g.hgt. mean anomaly

Note the approx. 3 wave pattern & the anomalously high amplitude of the wave pattern.

From: Rensch et. al.

So far…

• Geostrophy — ignoring time evolution. Gives basic features of synoptic scale - highs, lows etc. No divergence.
• Vorticity equation, potential vorticity conservation — Prognostic capability for developing highs, lows, expected weather. Divergence balanced by vertical motion.
• Rossby Waves — Steering of surface systems by rossby waves at 500hPa (LND).

NOW:
• Baroclinic instability (expand development ideas)
• Tropics? (completely different to mid latitudes - weak P gradients, convection through whole depth of atmosphere.)
Baroclinicity & thermal wind

- Baroclinicity will allow us to expand our ideas of system development & lead to wind variations with height.
- To consider baroclinic instability, first need to consider a baroclinic atmos. i.e:
- Density changes with temperature as well as with pressure $\Rightarrow$ thermal wind balance.

$$\begin{align*}
\frac{d\theta}{dz} &= \frac{\partial}{\partial z} \left( \frac{\rho_0 f dz}{\rho_f dz} \right) \\
\frac{d\theta}{dz} &= -\frac{\partial}{\partial z} \left( \frac{\rho_0 f dz}{\rho_f dz} \right)
\end{align*}$$

Physically...

- Geostrophic balance at each level to balance the different PGF.

$$\begin{align*}
\frac{\partial}{\partial z} \left( \frac{1}{\partial T} \right) &= \frac{\partial}{\partial z} \left( \frac{\rho_0 f dz}{\rho_f dz} \right)
\end{align*}$$

Let’s just consider the final part,

$$\frac{\partial}{\partial z} = \frac{\partial}{\partial z} \left( \frac{\rho_0 f dz}{\rho_f dz} \right)$$

If we put $\rho = \rho_0/RT$ and expand then we get,

$$\frac{\partial}{\partial z} = \frac{\rho_0 f dz}{\rho_f dz} \left( \frac{1}{\partial T} \right)$$

In strongly baroclinic areas the temperature gradients are much larger than the pressure gradients and we have

$$\frac{\partial}{\partial z} = \frac{\rho_0 f dz}{\rho_f dz} \left( \frac{1}{\partial T} \right)$$
**What is the wind shear?**

\[ u_T = \frac{\partial u}{\partial z} \]

From the ideal gas equation, \( T = \frac{p}{\rho} \) and so

\[ \frac{\partial T}{\partial z} = \frac{\rho \partial p}{\partial z} \frac{T}{\rho} \]

And from the hydrostatic approximation, \( \omega = \frac{p}{R} \) and so

\[ \eta_T = \eta_u(P_f) - \eta_u(P_i) = \frac{\partial}{\partial z} \int_P^{P_f} \Omega \cdot \frac{\partial P}{\partial T} \eta_T \eta_T \eta_T = R \frac{\partial \eta_T}{\partial P} \]

- \( T \) = thickness. Hence gauge thermal wind (and steering) from thickness charts.

**In the vicinity of a front,**
the temperature falls 10°C per 100 km. If the surface wind is 10km/h, what is the approx. predicted wind at 850hPa (5000 ft)?

At 34S, \( U(850) \sim U(\text{sfc}) + 205 \text{ km/h} \)

**In Baroclinic Instability,**

- Suppose the wind field is perturbed.
- Will lead to temperature anomalies like shown.
- Will only grow if there is available K.E.

From: Visconti, [7]

**Baroclinic Instability**

- Suppose the wind field is perturbed.
- Will lead to temperature anomalies like shown.
- Will only grow if there is available K.E.

In (a), any meridional movement conserves potential energy. In (b), meridional movement as shown decreases P.E. 
\[
\Rightarrow \text{Hence avail. for K.E. & disturbance can grow.}
\]
**Baroclinic instability**  
From: Visconti, [7]

We want $y << a$. Since $y$ is of order $O(\frac{\alpha T}{\alpha})$ and $a$ is of order $O(\frac{\alpha y}{\alpha})$, we need to scale for $\frac{\alpha y}{\alpha}$ to see when this will happen.

\[ \omega \approx \frac{\partial A}{\partial A} \approx \frac{(T+T)A}{T+T} \]

\[ \alpha \frac{\partial A}{\partial A} = \alpha \frac{\partial A}{\partial A} \]

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http://www.gfd-dennou.org/library/gfd Exp/exp_e/list/

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**Exercise:**

- Where is development most likely on the chart?

From [3]

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**Baroclinicity aids the vorticity**

The baroclinic term is:

\[ \alpha^2 J(\rho, p) \]

which is proportional to:

\[ (\nabla \rho \times \nabla T) \]

Temperature variations induce a circulation

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From Max Adams notes

Exercise:

- Where is development most likely on the chart?
Tropics

Tropical weather characterised by:

- Small variations in pressure (no geostrophy)
- Low diurnal temperature range.
- Monsoons.
- Cyclones.
- Thunderstorms

### Rossby number

- Defines relative importance of coriolis force
  \[ R_0 = \frac{\frac{du}{dt}}{\frac{1}{\rho} \frac{dp}{dx} + \frac{f}{u}} \]

  - For large scale motions - like gravity waves, Rossby number is small.
  - For the mid-latitudes where \(|f|\) large, \(R_0\) is also small.

Put \(t = \frac{L}{U}\) where \(L\) and \(U\) are horizontal length and velocity scales respectively. Then at 45°S, \(R_0 \approx 0.1\). At 10°S, \(R_0 \approx 0.4\).

**Wind field drives pressure field in tropics**

### Tropics

- There is a much smaller Coriolis effect near the tropics.

  \[
  \frac{du}{dt} = \frac{1}{\rho} \frac{dp}{dx} + \frac{f}{u} \\
  \frac{dv}{dt} = \frac{1}{\rho} \frac{dp}{dy} \\
  \frac{v}{\rho} \frac{dp}{dx} = \frac{1}{L} \frac{U^2}{L} = 10^4 \text{ m/s}^2
  \]

  Winds drive the pressure gradient in tropics, instead of pressure gradient driving winds.

  => Use streamline charts

\[ f = 10^{-4} \text{ m/s}^2 \]
Longer timescales - Waves

- Kelvin Waves
- Madden-Julian Oscillation
- Rossby Waves

Key driver of active & break periods of monsoon

- Tracked via OLR, 850 hPa and 200 hPa zonal wind anomalies.
- Also known as 40-50 day oscillation.
- Travels W to E at 5-10 m/s.
- Origin/physical mechanisms not well understood.

Kelvin Waves

Remember the shallow water equations (p replaced by h)

\[ \frac{d u}{d t} + \frac{1}{h} \frac{d}{dx} \left( h u \right) = 0 \]
\[ \frac{d v}{d t} + \frac{1}{h} \frac{d}{dy} \left( h v \right) = 0 \]
\[ \frac{d h}{d t} + \frac{1}{h} \left( u \frac{d h}{dx} + v \frac{d h}{dy} \right) = 0 \]

where \( H \) is the unperturbed depth of the fluid and we assume a barotropic fluid, i.e. pressure changes due only to depth changes.

Before: \( f=0 \), \( v=0 \) and \( d/dy \) all quantities = 0. Now \( d/dy \) is not zero and allow \( f=0 \)

Assume solutions like: \( u' = x g \exp[i(kx - \omega t)] \) and \( H' = h g \exp[i(kx - \omega t)] \)
Then
\[-i\alpha + g\beta = 0\]
\[f\beta + g\beta\frac{dy}{dx} = 0\]
\[-\alpha\beta + \beta\frac{d\beta}{dy} = 0\]

and
\[\frac{d\phi}{dy} = c^2 - gH\]
\[\frac{d\phi}{dx} = -g\frac{\phi}{\sqrt{gH}}\]

Kelvin Waves have been known to trigger El Nino’s (see )

Maximum div east of sfc low.
Moves east fast ~ 10m/s
Boundary is the equator.
Non-equatorial Kelvin waves occur near coastlines.

MJO

- 1971 Roland Madden and Paul Julian noticed periodic pattern in upper wind anomalies over the Pacific.
- Known as MJO. Driver of wet/dry spells in wet season.

Monitoring Kelvin W & MJO

- Madden-Julian Oscillation
  - Tracked via OLR, 850 hPa and 200 hPa zonal wind anomalies.
  - Also known as 40-50 day oscillation.
  - Travels W to E at 5-10 m/s.
  - Origin/physical mechanisms not well understood.

Key driver of active & break periods of monsoon
Cyclones

Intense tropical storms where winds average over 64 knots. In the absence of winds, a central pressure of 995 hPa is used. Warm cored. Different mechanisms at play near the centre and on the fringes.

\[ T = \frac{\partial P}{\partial z} = \frac{8000}{13} = 615 \]

at 5°N, 7°W. Hence, we can ignore the Coriolis force in the equations of motion.

\[ \frac{dT}{dt} = \frac{1}{\rho} \frac{d\rho}{dt} \]

Cyclostrophic balance

\[ \frac{1 - \rho^2}{\rho \Omega^2} = 10^{-4} \text{ m s}^{-2} \]

\[ \frac{dN}{dt} = \frac{1}{\rho} \frac{\partial \rho}{\partial t} \]

\[ \frac{dE}{dt} = \frac{1}{\rho} \frac{\partial \rho}{\partial t} \]

Near the centre, \( P \) changes by up to 10 hPa over just 50 km, with wind speeds over 4 m s\(^{-1}\). Hence, for a cyclone of central pressure 995 hPa, embedded in an ambient pressure of 1010 hPa, \( v_c \approx 39 \text{ m s}^{-1} = 78 \text{ kts} = 270 \text{ km h}^{-1} \).

\[ v_c \approx 39 \text{ m s}^{-1} = 78 \text{ kts} = 270 \text{ km h}^{-1} \]

which is hurricane force.

If central pressure is only 925 hPa (Thelma), \( 1050 \text{ km h}^{-1}! \)

Empirical, Method gives 205 km h\(^{-1}\). Real winds were \(-350 \text{ km h}^{-1}\).

In the Australian region, \( C = 2.5, x = 0.7 \) (Crane 1985)

Hence, for cyclone of central pressure 955 hPa, embedded in an ambient pressure of 1010 hPa, \( v_c \sim 39 \text{ m s}^{-1} = 78 \text{ kts} = 270 \text{ km h}^{-1} \).

which is hurricane force.

Thunderstorms and stability

.... next time.
References


[8] Special Climate Statement 17: The exceptional January-February heatwave in south eastern Australia. Melbourne Centre, Bureau of Meteorology


exp/gfd_exp/exp_e/doc/bc/guide01.htm