

DERIVATIONS IN ALGEBRAS OF OPERATOR-VALUED FUNCTIONS

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ABSTRACT

It is well known that any derivation acting on a von Neumann algebra is inner. In particular, there are no non-trivial derivations on a commutative von Neumann algebra $\mathcal{M} = L_\infty(0, 1)$.

Consider an arbitrary semifinite von Neumann algebra \mathcal{M} and the algebra $S(\mathcal{M})$ of all measurable operators affiliated with \mathcal{M} (the algebra $S(\mathcal{M})$ was first introduced by I.E. Segal in 1953 and is a cornerstone of noncommutative integration theory). Recently, there have appeared a number of publications treating the question whether every derivation on $S(\mathcal{M})$ is inner.

In 2006, it was established by A.Ber, V. Chilin and F.S. that if $\mathcal{M} = L_\infty(0, 1)$, then the algebra $S(\mathcal{M})$ (which in this case may be identified with the algebra $S(0, 1)$ of all Lebesgue measurable functions on $(0, 1)$) has non-trivial derivations, in particular, the classical derivations d/dt extends to $S(0, 1)$. Our main objective in this talk is to present a recent result by A. Ber (Tashkent), B. de Pagter (Delft) and F.S. that if $\mathcal{M} = L_\infty(0, 1) \bar{\otimes} B(H)$ (that is \mathcal{M} is a von Neumann tensor product of the algebra $L_\infty(0, 1)$ and the algebra $B(H)$ of all bounded linear operators on an infinite dimensional separable Hilbert space), then every derivation on $S(\mathcal{M})$ is still inner.