Analysis Seminar talk (SMS, UNSW)

A transformation of almost periodic pseudodifferential operators to Fourier multiplier operators on vector-valued functions.

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Abstract We treat pseudodifferential operators on $\mathbf{R}^d$ in the Kohn–Nirenberg quantization, where the symbol $a(\cdot,\xi)$ is almost periodic (a.p.) for each $\xi \in \mathbf{R}^d$, and belongs to a Hörmander class $S^m_{\rho,\delta}$. We study the symbol transformation $a \mapsto U(a)$

$$U(a)(\xi)_{\lambda,\lambda'} = M_x(a(x,\xi-\lambda')e^{-2\pi i x(\lambda'-\lambda)})$$

where M_x denotes the mean value for a.p. functions, which was introduced, for operator kernels rather than symbols, by E. Gladyshev. $U(a)(\xi)$ can be considered a matrix indexed by $(\lambda,\lambda') \in \Lambda \times \Lambda$ where Λ is the set of frequencies that occur in $\{a(\cdot,\xi)\}_{\xi\in\mathbf{R}^d}$. Thus $U(a)$ may be considered the operator-valued symbol of a Fourier multiplier operator that acts on vector-valued functions.

Using results by M. A. Shubin, we show that the transformation respects operator composition, $U(a\#_0 b)(\xi) = U(a)(\xi)\cdot U(b)(\xi)$, where $a(x,D)\circ b(x,D) = (a\#_0 b)(x,D)$. Moreover, $a(x,D) \geq 0$ if and only if $U(a)(D) \geq 0$. Positivity and boundedness on Besicovitch-Sobolev spaces of $a(x,D)$ are encoded in the matrix $U(a)(0)$.