Determining lower bounds on the maximal p-negative type of finite metric trees

Negative type inequalities arose classically (in the 1920s and 1930s) in connection with the study of isometric embeddings. Prime movers were Karl Menger and Isaac Schoenberg. More recently, negative type inequalities have been intensively studied in relation to certain *hard* (as in NP-hard, etcetera) optimization problems that arise in (or land in) the domain of theoretical computer science. Questions concerning hypercube embeddings are prominent in this direction.

Typically, it is a rather difficult nonlinear problem to determine meaningful lower bounds on the maximal p-negative type of any given metric space (X, d). And, rarely is that maximum actually computed. In this talk I will describe recent joint work with Ian Doust wherein we obtained meaningful lower bounds on the p-negative type of finite metric trees. Meaningful in this context means $p > 1$. The strategy we developed, which is based on the notions of *strict* negative type and *generalised* roundness, led to the isolation of an unexpected new family of inequalities for finite metric trees. We refer to the members of this family as inequalities of *enhanced negative type.* They feature the introduction of a positive additive constant to the left hand side of the classical negative type inequalities and are therefore considerably stronger. An *algorithmic* characterisation of when equality occurs in these enhanced inequalities will also be described, time and chalk permitting.

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