Proceedings of the Edinburgh Mathematical Society (2007) **50**, 509–510 ©
Printed in the United Kingdom

BOOK REVIEWS

DOI:10.1017/S0013091507215020

Wildberger, N. J. *Divine proportions* (Wild Egg Books, 2005), xx + 300 pp., 0 975 74920 X (hardback), $109.95 (AUS).

Remarkable claims are made in this book. Essentially self-published (although of high quality and by an academic mathematician) it promises to 'challenge and change your understanding of mathematics'. The claims are these: first, that the geometrical methods presented, which abolish the notions of angle and distance, are a better and more natural way to do classical trigonometry; second, that the methods are more powerful because they work over a general field; and, finally, that the methods are more rigorous because they avoid certain foundational issues.

The main idea is that 'geometry is a quadratic subject'. Distance is replaced by 'quadrance' and angle by 'spread', which is defined as a ratio of quadrances. These quantities turn out to be the square of distance and the square of the sine of angle, respectively, although the definitions given need only the concepts of incidence and perpendicularity, and are expressed entirely as rational functions in Cartesian coordinates. The price paid is that one can no longer add these new notions of distance and angle linearly.

Part I is discursive and stakes out the claims more fully. Next follows a rigorous development of the basics of rational trigonometry. Part III, entitled 'Universal geometry', takes this further, to handle more intricate properties of triangles and to introduce conics and various other curves. The final part demonstrates practical uses, including physics and surveying, and looks ahead towards higher dimensions and polar coordinates.

The evidence for at least two of the claims is impressive. With regard to the first, the author calls attention to the 'wrongness' of invoking transcendental functions associated with infinite sums and the completeness of the reals in order to solve elementary problems about triangles. His approach, in contrast, will for integer-valued problems always give answers that are rational or involve square roots at most. Thus computations are exact and do not require a calculator. His remark that with the traditional approach 'students are constantly given examples that deal with $90^\circ/60^\circ/30^\circ$ or $90^\circ/45^\circ/45^\circ$ triangles since these are largely the only ones for which they can make unassisted calculations' must ring true for all who have ever set an exam. Moreover, spread does not distinguish between an angle and its supplement, thereby eliminating a common source of student confusion. The author even dares to compare the relation between traditional trigonometry and his proposal with the relation between cumbersome Roman numerals and the modern Arabic notation. While this may be going a little too far, the fact that all ordinary problems in plane geometry can be solved simply and exactly by hand in this way is undeniable.

The second claim, that this is a universal kind of geometry not tied to the reals, is also impressive. All definitions and theorems are stated in generality and work over any field not of characteristic two. Seeing the classical results about circles and chords treated in this way, pictures of cardioids and lemniscates over finite fields, and even tangents to conics without calculus is, for the current reviewer at least, something of a revelation.

The third claim, that there are foundational flaws in classical geometry, is less convincing. It centres around the absence of a definition of angle, and some concerns over the status of $\mathbb{R}$ with regard to computability. The author admits that these are minority views of his, and does not devote much space to them.

Despite the attractive cover, this is not a popular-mathematics book and is unfortunately not an easy read. On randomly flipping it open one meets technical-looking diagrams and long formulae. There are few prerequisites, but the stated audience is mathematicians and scientists, although 'teachers with a strong interest' and 'gifted undergraduates' should 'be able to follow'. It demands to be read with pen and paper, as there are over 200 exercises interspersed, and it could perhaps be used as a course textbook were a course on plane geometry over arbitrary fields desired. The coverage is comprehensive—many of the results will be unfamiliar to the reader even in the Euclidean case—but it is largely the emphasis on finite fields that increases the difficulty. Readers struggling to become familiar with quadrance and spread must also contend with matters such as two lines being simultaneously parallel and perpendicular, and with what would ordinarily be a simple inequality becoming the thornier issue of whether or not a certain expression is a square.

Moreover, beyond the introductory part, the author has chosen clarity and rigour at the expense of expository discussion. Perhaps this is understandable; he may feel that he must defend his controversial position by making the mathematics logically impeccable; but while it succeeds in this it becomes dry for the reader. Some helps are provided, however: clear layout with important statements shaded; all theorems named as well as numbered, and listed at the back; and numerous diagrams on which quadrances and spreads are labelled with a well-chosen notation. Probably in a deliberate attempt to shake the reader out of ingrained thinking, the author seldom mentions the corresponding parts of classical trigonometry, nor even explains in words what each formula is really saying. Thus readers must decode for themselves that, for example, the 'Spread Law' corresponds to the sine rule. Clues are provided, however: it is no accident that the angle-related quantities called spread, cross and twist turn out to be the squares of sin, cos and tan.

There are some real gems in this book, especially towards the end, and it is a pity that only the most motivated reader will reach them. They include Platonic solids, all of whose face spreads turn out to be rational; the Chebyshev polynomials and their role in symmetry, including the fact that $\mathbb{F}_{19}$ is the smallest field containing regular pentagons; and an unusual and beautiful solid convex body called an ellipson.

One criticism (perhaps owing to ignorance on the part of the reviewer, who is not a geometer) is that it is not clear whether any or all of the theorems are new. A few are attributed; for the rest, perhaps one is to assume that the Euclidean result is ancient and the universal one Wildberger's own.

Overall, the book gives plenty of food for thought in the early chapters, while the parts at the end reward the effort spent getting there. That this *is* the right way to do geometry seems amply justified, and the question now is whether it can overcome the barriers to change. It deserves serious attention both from mathematicians and from educationalists. For the former, a further book that extends the methods to spherical and hyperbolic geometries is promised; while for the latter, a more elementary treatment is hinted at, and there is already some material aimed at schools on the author's website. A revolution in the way trigonometry is taught sounds virtually impossible—indeed, even the reviewer, who wants to adopt the approach for her day-to-day calculation, is finding that old habits die hard. However, mathematicians should read this out of curiosity and watch future developments with interest.

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