A general approximation scheme for
attractors of abstract parabolic problems

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In this talk we consider the semilinear problems of the form $u' = Au + f(u)$, where $A$ generates an exponentially decaying compact analytic semi-group in a Banach space $E$ and $f$ is globally Lipschitz and bounded map from $E^\alpha$ into $E$ ($E^\alpha = D((-A)^\alpha)$ with the graph norm). These assumptions ensure that the problem has a global attractor. Under a very general approximation scheme we prove that the dynamics of such problem behaves upper semicontinuously.

We also show that, if all equilibrium solutions of this problem are hyperbolic, then there is an odd number of such equilibrium solutions. Additionally, if we impose that every global solution converges as $t \to \pm \infty$, (e.g. gradient semigroups with isolated equilibria), then we prove, under this approximation scheme, that the attractors also behave lower semicontinuously. This general approximation scheme includes the finite element method, projection and finite difference methods.